CSCI 4360/6360 Data Science II

Linear Dynamical Systems

Last time…

- Motion analysis via optical flow
- Parametric vs energy-based formulations
- Importance of assumptions
- Modern formulations
  - Robustness to outliers (large optical flow)
  - Relatedness to markov random fields
  - Coarse-to-fine image pyramids

Today

- A specific type of motion: **dynamic textures**

Dynamic textures

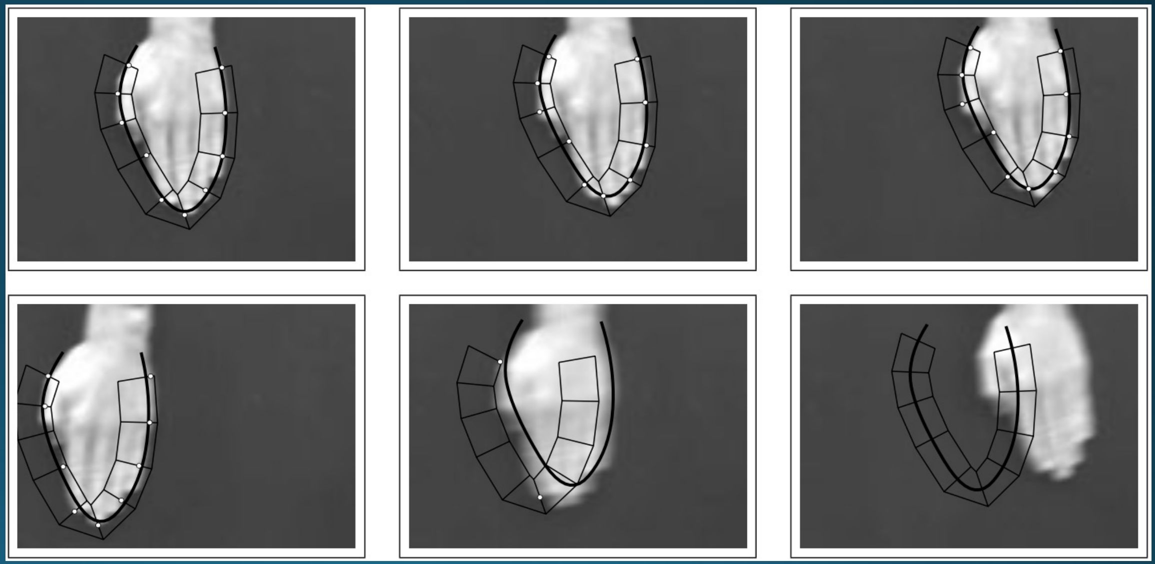
- *"Dynamic textured sequences are scenes with complex motion patterns due to interactions between multiple moving components."*
- Examples
  - Blowing leaves
  - Flickering flames
  - Water rippling
- **Multiple moving components: problematic for optical flow**
- How to analyze dynamic textures?

Dynamical Models

- Goal: an effective procedure for tracking changes over sequences of images, while maintaining a certain coherence of motions

Dynamical Models

- Hand tracking
- Top row: slow movements
- Bottom row: fast movements
- **Fixed curves or priors cannot exploit coherence of motion**



Linear Dynamical Models

- Two main components (using notation from Hyndman 2006):

Appearance Model

$$y_t = Cx_t + u_t$$

State Model

$$x_t = Ax_{t-1} + Wv_t$$

Autoregressive Models

- This is the definition of a 1st-order autoregressive (AR) process!

$$x_t = Ax_{t-1} + Wv_t$$

- Each observation (x_t) is a function of previous observations, plus some noise

- **Markov model!**

Autoregressive Models

- AR models can have higher orders than 1
- Each observation is dependent on the previous *d* observations

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \ldots + A_d x_{t-d} + W v_t$$

Appearance Model

- y_t: image of height *h* and width *w* at time *t*, usually flattened into 1 x *hw* vector
- x_t: state space vector at time *t*, 1 x *q* (where *q* <<< *hw*)
- u_t: white Gaussian noise
- *C*: output matrix, maps between spaces, *hw* x *q*

Output matrix

$$y_t = Cx_t + u_t$$

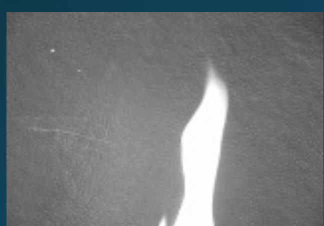
Image in a sequence

Low-dimensional "state"

Noise inherent to the system

Appearance Model

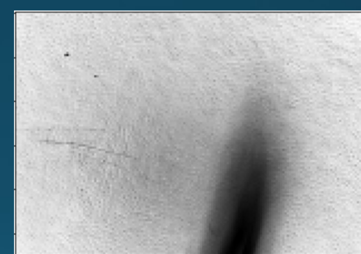
$$y_t = Cx_t + u_t$$



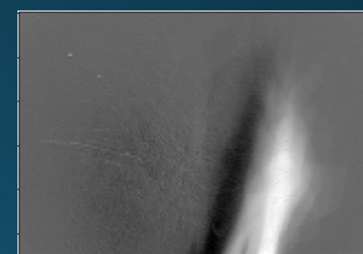
Each of these is 1 column of *C*.

There are *q* of them (first 4 shown here).

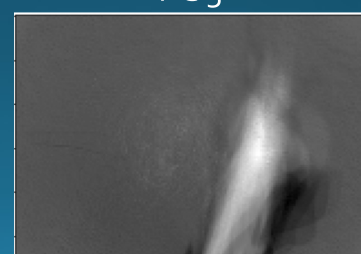
PC 1



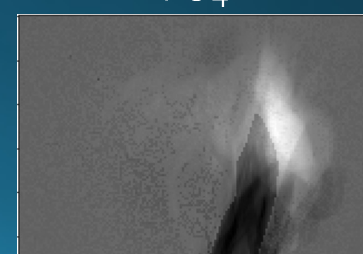
PC 2



PC 3



PC 4



Appearance Model

- How do we learn the appearance model?
- Choose state-space dimension size *q*
- Noise term is i.i.d Gaussian

$$y_t = Cx_t + u_t$$

U-hat is a matrix of the first *q* columns of *U*

$$Y = [\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_f]^T$$

$$Y = U\Sigma V^T$$

$$C = \hat{U}$$

$$X = \hat{\Sigma}\hat{V}^T$$

V-hat is a matrix of the first *q* columns of *V*, and sigma-hat is a diagonal matrix of the first *q* singular values

State Model

- x_t and x_{t-1}: state space vectors at times t and $t-1$, each 1 x q vector
- A: transition matrix, q x q matrix
- W: driving noise, q x q matrix
- v_t: white Gaussian noise

State transition matrix

$$x_t = Ax_{t-1} + Wv_t$$

Low-dimensional state at time t

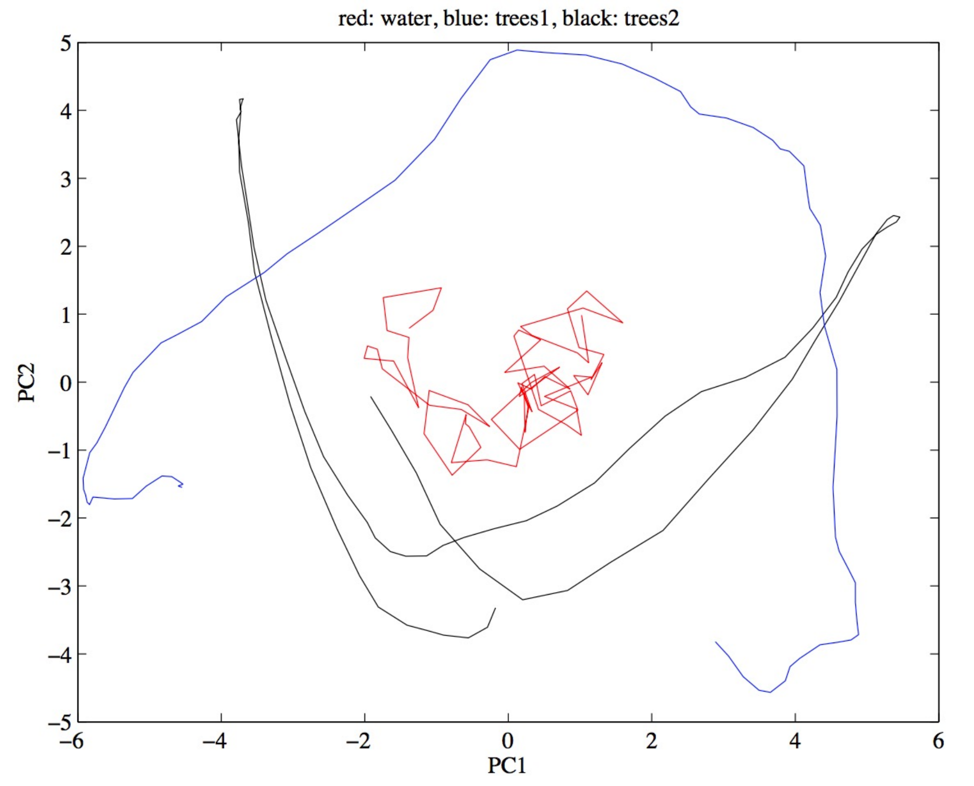
Low-dimensional state at t - 1

Driving noise

State Model

$$x_t = Ax_{t-1} + Wv_t$$

- Three textures
- *q* = 2



State Model

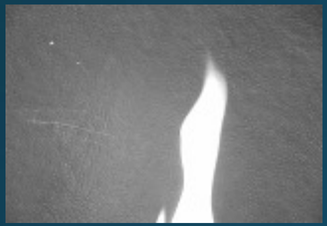
- How do we learn the state model?

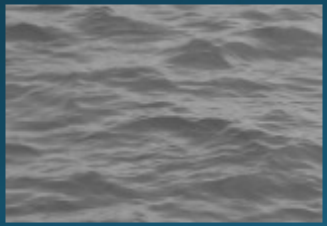
$$x_t = Ax_{t-1} + Wv_t$$

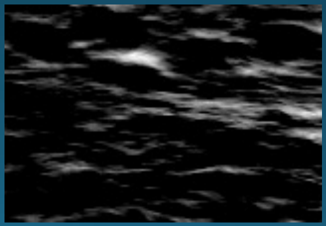
- **Stay tuned for Homework 2…**

LDS as Generative Models

- Once we've learned the parameters, we can *generate new instances*







- Major strength of LDS!

Problems with LDS

- PCA = Linear + Gaussian
- What if the *state space* isn't linear, or data aren't Gaussian?

- Nonlinear appearance models
  - Wavelets
  - IsoMap
  - LLE
  - Kernel PCA
  - Laplacian Eigenmaps

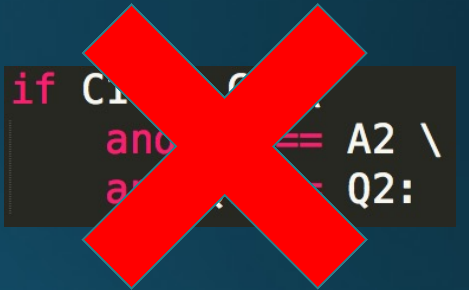
- These introduce their own problems!

Problems with LDS

- Comparing LDS models

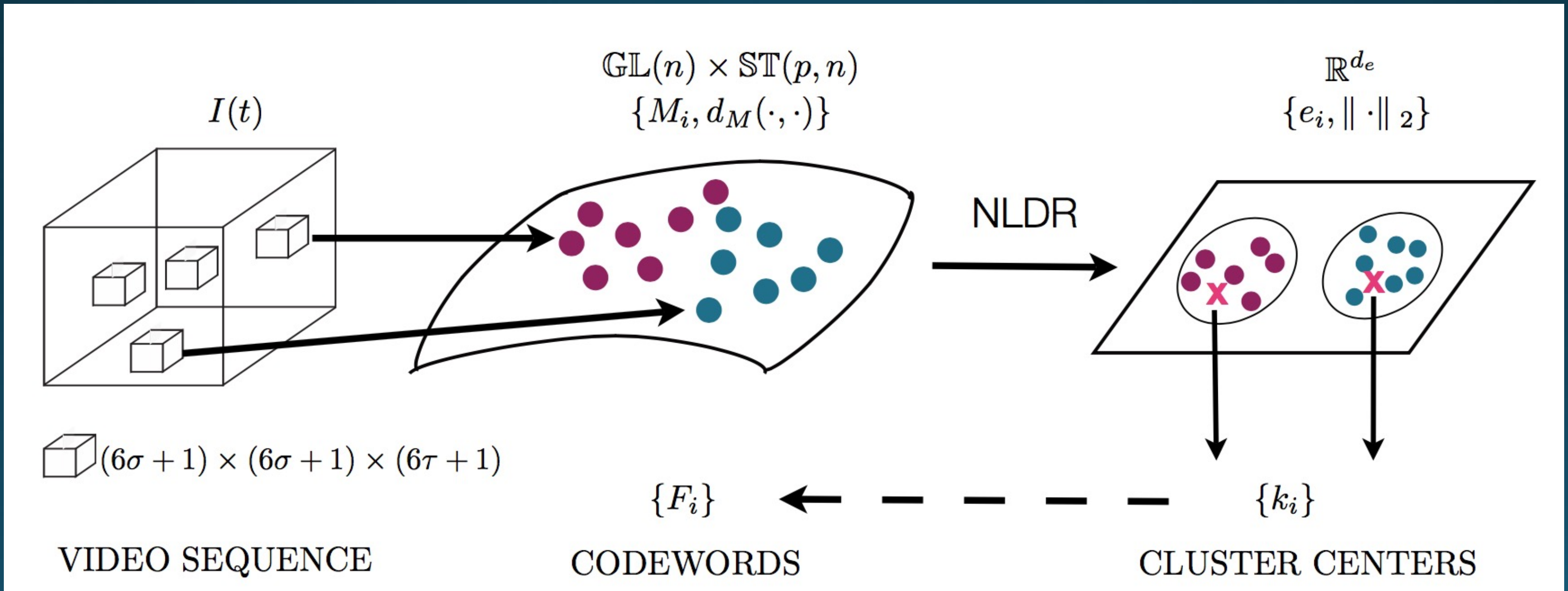
- Given a sequence *Y*:
- New sequence *Y'*:
- **How do we compare these systems?**
- Despite linear formulation, θ are NOT Euclidean
- Valid distance metrics include spectral methods and distribution comparators

$$\theta = (C, A, Q)$$
$$\theta' = (C', A', Q')$$



Comparing LDS

- Select multiple, non-overlapping patches from each video
- Build LDS for each patch



Conclusion

- Dynamic textures are motion with statistical regularity
- Regularity can be exploited through parametric representation
- Linear dynamical systems (LDS)
  - Autoregressive models (AR)
  - Markov assumption
  - Representation model + State model
  - Generative models

References

- Hyndman *et al*, "Higher-order Autoregressive Models for Dynamic Textures", BMVC 2007 http://www.cs.toronto.edu/~fleet/research/Papers/DynamicTextures.pdf
- Shumway and Stoffer, *Time Series Analysis and Its Applications* (3rd ed.), Chapters 3 and 6, http://www.db.ucsd.edu/static/TimeSeries.pdf
- Blake and Isard, *Active Contours*, Chapter 9-11, http://ww.vavlab.ee.boun.edu.tr/courses/574/material/books/blake_Active_Contours.pdf
- Doretto *et al*, "Dynamic Textures", IJCV 2003 https://escholarship.org/uc/item/2m50f2fb
- Woolfe *et al*, "Shift Invariant Dynamic Texture Recognition", ECCV 2006 https://link.springer.com/chapter/10.1007%2F11744047_42?LI=true
- Ravichandran *et al*, "View-invariant dynamic texture recognition using a bag of dynamical systems", CVPR 2009 http://ieeexplore.ieee.org/abstract/document/5206847/

Homework 1 Questions?